Probability Concepts.

① If all outcomes in the sample space are equally likely: For event A: $P(A) = \frac{\#(A)}{\#(S)}$;

Note that this also applies if we're looking at the results of a sequence of events with each event having equally likely outcomes.

② Permutation: $P[n,r] = \frac{n!}{(n-r)!}$; Combination: $C[n,r] = \frac{n!}{r!(n-r)!} = \frac{1}{r!}P[n,r]$;

③ If we have an experiment with a constant probability of "success" p and that experiment is repeated n times such that each trial is independent of the other trials, we can use the Binomial Prob. Model: $P(r \text{ "successes"}) = C[n,r](p)^r(1-p)^{n-r}$;

④ Conditional Probability: $P(A|B) = \frac{P(A\cap B)}{P(B)}$;

⑤ Compound Probability. AND: $P(A\cap B) = P(A)\cdot P(B|A)$; $P(A\cap B) = P(B)\cdot P(A|B)$; OR: $P(A\cup B) = P(A) + P(B) - P(A\cap B)$;

⑥ Independence: The following are equivalent: (1) A and B are independent;
(2) $P(A|B) = P(A)$ and $P(B|A) = P(B)$;
(3) $P(A\cap B) = P(A)\cdot P(B)$;

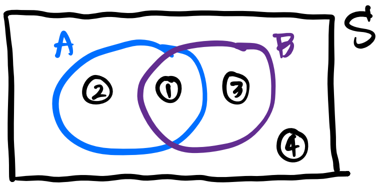
⑦ Mutual Exclusiveness: The following are equivalent: (1) A and B are mutually exclusive.
(2) $P(A\cap B) = 0$.
(3) $A\cap B = \phi$.
(4) $P(A\cup B) = P(A) + P(B)$.

⑧ Complement: For an event A: $P(A) + P(\bar{A}) = 1$
$P(A|B) + P(\bar{A}|B) = 1$ for some event B.

⑨ Partition Theorem: If we have a partition of $S = B_1 \sqcup B_2$ (with B_1 and B_2 mutually exclusive): then for all events A: $P(A) = P(A\cap B_1) + P(A\cap B_2)$;
A common partition is $S = A \sqcup \bar{A}$ for any event A.

⑩ In a Venn diagram with 2 events A and B, there is a partition of S by $2^2 = 4$ events.
That is, $S = (A\cap B) \sqcup (A\cap \bar{B}) \sqcup (\bar{A}\cap B) \sqcup (\bar{A}\cap \bar{B})$.

Illustrated:



$A\cap B$: ①
$A\cap \bar{B}$: ②
$\bar{A}\cap B$: ③
$\bar{A}\cap \bar{B}$: ④

Since we have a partition, we have: $P(S) = 1 = P(A\cap B) + P(A\cap \bar{B}) + P(\bar{A}\cap B) + P(\bar{A}\cap \bar{B})$;

⑪ Bayes' Theorem: $P(A|B) = \frac{P(B|A)\cdot P(A)}{P(B)}$;